Modeling of a Self-Oscillating Cantilever

James Blanchard, Hui Li, Amit Lal, and Douglass Henderson
University of Wisconsin-Madison
1500 Engineering Drive
Madison, Wisconsin 53706

Abstract

A radioisotope-powered, self-oscillating cantilever beam has been developed for small scale applications. A thin beam is placed within a short distance from a radioisotope source and as the charged particles from the source are collected on the beam, it is attracted towards the source. As it contacts the source, the beam is discharged and returns to it's initial position. The period of oscillation is governed by the time it takes for the beam to contact the surface and discharge. A model has been developed to provide understanding of the behavior of such a device, and to help explore potential applications. Based on a single dimensionless parameter, this model predicts the design space for which the beam will self-oscillate, as well as regimes for which contact is never achieved. Initial benchmarks of the model are encouraging.

Introduction

Efficient on-board power for MEMS devices will create an opportunity for a wide variety of applications. Previously suggested power sources include fossil fuels, fuel cells, chemical batteries, and solar energy, but nuclear power sources provide significant advantages for applications requiring long lives or high power density. Such sources have been extensively researched on fairly large scales [1-4], but application to the microscale is just beginning. One approach to harnessing radioisotope power for MEMS devices is to collect radiated charges across a capacitor. This is called a direct conversion nuclear battery [2]. Other options utilize the heat produced by radioactive decay, along with thermoelectric or thermionic devices to produce electricity. In this paper we demonstrate a novel direct conversion battery in which one of the electrodes is elastically deformable. Recently [5], feasibility and a preliminary model were presented. In this paper we present an analytical model that results in an elegant approach to understanding various areas of oscillator operation.

Potential for Isotope-Powered MEMS devices

Some typical isotopes that one would use in a nuclear powered MEMS device are presented in Table 1. Characteristics required for these applications include low range (to avoid passing through the device) and an absence of gamma emission (for safety reasons). The 210-Po isotope in this table is an alpha emitter, but all the others are beta emitters and none exhibit gamma emission. Power densities range from 0.006 to 137 W/g. The lifetime of these devices will depend on the half-life of the isotope, so one can easily get a battery which retains a substantial fraction of it's available power over decades.

One way to assess the life of a power source is to compare the energy density, which integrates the power over the life (without recharging). A typical chemical battery has an energy density on the order of 1 kJ/g, while a typical nuclear battery will have contain well over 10,000 kJ/g. Hence, power sources fueled by radioisotopes are ideal for applications requiring high power density and a long life (without refueling). The experiments described below employ 63-Ni to power an oscillator.

Isotope	Average energy	Half life	Specific activity	Specific Power	Estimated Range in Cu
	(KeV)	(year)	(Ci/g)	(W/g)	(microns)
63-Ni	17.4	100.2	57	0.006	14
90-Sr	195.8	28.8	138	0.16	332
3-H	5.7	12.3	9664	32.5	3
210-Po	5304.3	0.38	4493	137	11
32-P	694.9	0.04	285700	1.18	1344

TABLE 1. Some candidate radioisotopes for MEMS applications. The first three columns are obtained from Reference 6.

A Nuclear Powered Oscillating Actuator

A novel application of radioisotope power at small scales is the realization of a self-reciprocating or oscillating actuator that can generate forces for microscale systems. The central idea behind this oscillator is to collect the charged particles emitted from the radioisotope by a cantilever. By charge conservation, the radioisotope thin film will have opposite charges left as it radiates electrons into the cantilever. Thus an electrostatic force will be generated between the cantilever and the radioisotope thin film. The resulting force attracts the cantilever toward the source. With a suitable initial distance the cantilever eventually reaches the radioisotope and the charges are neutralized via charge transfer. Although the exact mechanisms of charge transfer can be tunneling or direct contact, the time scale of the charge transfer is much shorter than the reciprocation cycle, allowing the details to be ignored for cantilever performance analysis. As the electrostatic force is removed, the spring

force on the cantilever retracts it back to the original position and it begins to collect charges for the next cycle. Hence, the cantilever acts as a charge integrator allowing energy to be stored and converted into both mechanical and electrical forms. Based on this idea, a prototype cantilever device has been made and an analytical model is developed. A schematic of the device is shown in figure 2.

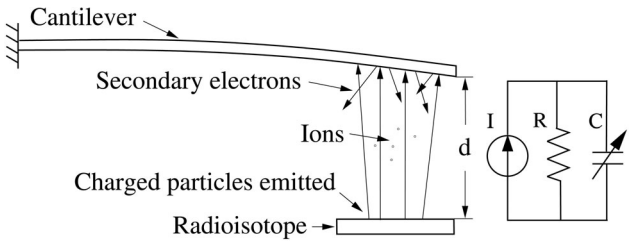

Figure 2: Schematic of the oscillator

Electromechanical Model

A model has been developed to provide understanding of the behavior of this oscillator [5]. The radioisotope source is modeled as a current source. The cantilever/source gap is modeled as a time varying capacitor. A parasitic resistor is included to model possible leakage paths for the collected charge. Several physical mechanisms may contribute to this resistance. Both naturally occurring ions, and ions created by electronic collisions between emitted particles and gas molecules will constitute a current. Furthermore, secondary electrons emitted from the cantilever due to high-energy electron-substrate collisions may contribute to the leakage current with a polarity opposite to the emitted current. Charge conservation results in:

$$\alpha I - \frac{V}{R} - \varepsilon_0 A \frac{\partial}{\partial t}\left(\frac{V}{d}\right) = 0 \qquad (1)$$

where I is the total emitted current from the radioisotope, A is the area of the capacitor, R is the equivalent resistance, V is the voltage across the source and the cantilever, t is the time, d is the distance between the electrodes, and α is an empirical coefficient describing the portion of the total emitted current that gets collected by the cantilever. The first term is the emitted current; the second is the leakage current and the third is the displacement current. There are at least three reasons for imperfect charge collection (i.e., $\alpha < 1$). First, the charged particles emitted from the source have an angular distribution and only the particles that fall in the solid angle formed by the intersection of the source and cantilever are collected. Second, some high energy particles can travel through the cantilever. Third, when secondary electrons are emitted from the cantilever, positive charges are left in the cantilever, reducing the net negative charges.

The third term in equation1 is the displacement current of the capacitor. The electrical field E between the source and the cantilever has been approximated as uniform, i.e. E = V/d, because the angle of approach between the cantilever and the source is small allowing the approximation that an average gap d exists between the cantilever and the source.

Assuming that the cantilever moves very slowly, an assumption that is verified by experiment, one can ignore the cantilever's inertia. In this quasi-static approximation, the spring force of the cantilever exactly balances the electrostatic attraction force acting on the cantilever. This can be written as:

$$k(d_0 - d) = QE \qquad (2)$$

where k is the spring constant, d0 is the initial distance, d is the distance between the cantilever and the source, Q is the total charges on the cantilever and E is the electric field. Assuming a uniform electric field, the capacitor can be modeled as a parallel plate capacitor C and the charge on it is:

$$Q = CV = \frac{\varepsilon_0 A V}{d} \qquad (3)$$

Combining Equations 2 and 3 with the uniform electric field approximation gives:

$$k(d_0 - d) = \varepsilon_0 A\left(\frac{V}{d}\right)^2 \qquad (4)$$

which can be rewritten as:

$$V = \sqrt{\frac{k}{\varepsilon_0 A}}\sqrt{d_0 - d} \qquad (5)$$

Substituting Equation 5 into Equation1 results in:

$$\frac{\partial d}{\partial t} = \frac{2}{\varepsilon_0 RA}(d_0 - d)d - \frac{2\alpha I}{\sqrt{\varepsilon_0 kA}}\sqrt{d_0 - d} \qquad (6)$$

The behavior of the oscillator can be understood through a stability analysis of this equation. Before carrying out this analysis, it is convenient to introduce a few relevant dimensionless variables. With the following definitions:

$$u = \frac{d}{d_0}$$
$$\eta = \frac{2td_0}{\varepsilon_0 RA} \qquad (7)$$

equation 6 becomes

$$\frac{du}{d\eta} = u(1-u) - z\sqrt{1-u} \qquad (8)$$

where

$$z = \frac{\alpha I R}{d_0^{3/2}} \sqrt{\frac{\varepsilon_0 A}{k}} \tag{9}$$

Hence, the oscillator behavior is controlled by this single dimensionless parameter z. The significance of this parameter can be understood as the ratio of two currents, as shown below. If there is large resistance between the beam and the source, then the first term on the right side of equation 6 can be ignored and the solution is

$$\eta = \frac{2}{z}\sqrt{1-u} \tag{10}$$

From this solution, the time it takes for the beam to contact the source, found by setting u=0, is

$$\eta_{contact} = \frac{2}{z} \tag{11}$$

or

$$T = \frac{\sqrt{\varepsilon_0 A k d_0}}{\alpha I} \tag{12}$$

where T is the time to contact. If such a beam were fixed, the charge that accumulated on the beam during time T would produce a potential given by

$$V = \frac{Q}{C} = \sqrt{\frac{k d_0^3}{\varepsilon_0 A}} \tag{13}$$

The leakage current then is just V/R. The ratio of the current reaching the collector beam to this leakage current is the dimensionless parameter z. In this sense, z is a competition between the leakage and storage currents, with large z indicating low leakage currents. Hence, one would expect the beams to work more efficiently for large values of z.

The equilibrium points of equation 8 are found by setting the time derivative in equation 8 to 0 and solving for the roots of the resulting function. There is always an equilibrium point at u=1 and a second real root which is negative. This latter root will be ignored for the remainder of this discussion. For

$$z > \frac{2}{\sqrt{27}} \approx 0.38 \tag{14}$$

the remaining two roots are imaginary and the beam will always contact the source because the time derivative is always negative on 0<u<1. For z<0.38 there are two real roots between 0 and 1. The larger of these roots is a stable root, while the smaller is unstable. Hence, the beam will only be attracted to the source if the beam is somehow externally deflected to a dimensionless gap less than that given by the smaller root. By taking a Taylor series expansion in z of this lower root, one finds that the smaller root is approximately

$$u \approx z + \frac{z^2}{2} + \frac{5z^3}{8} \tag{15}$$

To summarize, the beam will not self-oscillate for z<0.38 and will only contact the source if u is externally decreased to the value given in equation 15. The value of this lower root is shown in Figure 3 (using the full solution, rather than just the approximation).

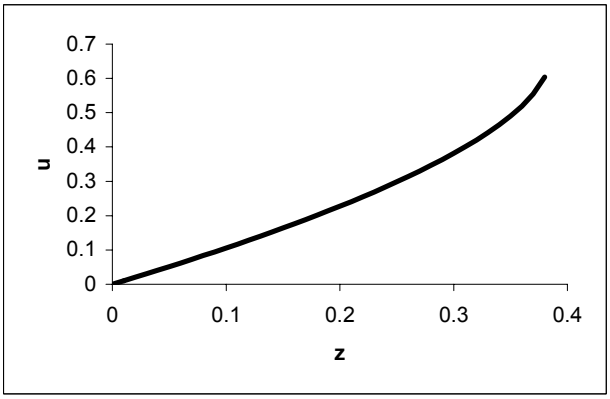

Figure 3: Smaller root of the right hand side of equation 8 for small values of z. The beam will not contact the source unless an external force reduces the gap to this value (or below).

A Variation on the Original Model

To the extent that the resistance R depends on the ion density between the beam and the source, one might expect R to be proportional to the gap width d. This changes the nature of the equilibria discussed in the previous paragraph. To explore this, we repeat the analysis after substituting

$$R = \rho d \tag{16}$$

into equation 6. In this equation, d is the instantaneous gap between the beam and the source and ρ is a constant. Physically, ρ represents the resistance per unit gap width between the source and the beam. Using equation 16, the dimensionless variables become

$$u = \frac{d}{d_0}$$
$$\eta = \frac{2t}{\varepsilon_0 \rho A} \tag{17}$$
$$w = \alpha I \rho \sqrt{\frac{\varepsilon_0 A}{k d_0}}$$

and the differential equation becomes

$$\frac{du}{d\eta} = (1-u) - w\sqrt{1-u} \qquad (18)$$

We now have a new dimensionless parameter w, which controls the behavior of the system. This system still has an equilibrium point at u=1. If w>1, there is no equilibrium point on 0<u<1, so the beam will always work. If w<1, then there is an equilibrium point at $u=1-w^2$. This is a stable point, so the beam will always deflect to this point and stop. It will never contact the source.

Equation 18 can be solved analytically. The time dependent solution for the gap is given by

$$u = 1 - w^2\left(1 - e^{-\eta/2}\right)^2 \qquad (19)$$

This result is plotted for three representative values of w in Figure 4.

From equation 19, we can determine the time to contact for the beam. For w<1, there is never contact. But for w>1, we find equation 20, which is plotted in Figure 5.

$$\eta_{contact} = -2\ln\left(1 - \frac{1}{w}\right) \qquad (19)$$

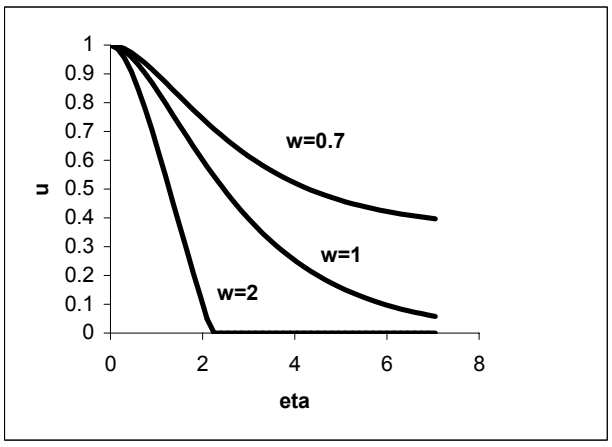

Figure 4: Dimensionless gap width vs dimensionless time for three values of w. Negative values of u have been set to 0.

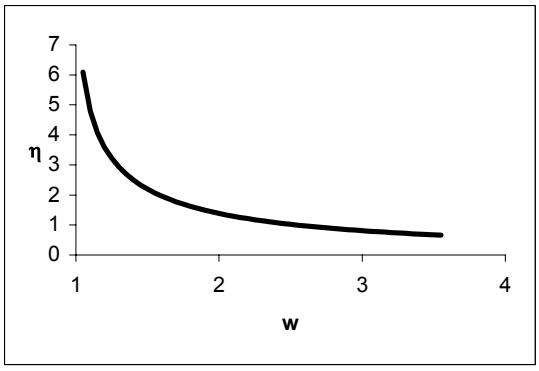

Figure 5: Dimensionless contact times as a function of w.

Prototype Device

A prototype device has been made to verify the cantilever operation and the model [5]. A β source made of ^{63}Ni is used as the radioisotope. The ^{63}Ni is electroplated as a 4 mm x 4 mm thin film on a 1 mm thick Al plate and the activity is 1 mCi. The cantilever is made of copper with dimensions 5 cm x 4 mm x 60 microns. The thickness 60 microns was chosen to capture most of the electrons as the penetration depth of a 67 KeV electron in copper is approximately 14 microns [7].

The cantilever is clamped between two nonconductive Teflon blocks for electrical insulation. The source is clamped by two glass slides, which are mounted on a Teflon base. The Teflon base is in turn mounted on a linear motion stage used to control the initial distance between the source and the cantilever. The setup is placed inside a vacuum chamber with a glass top. A microscope connected to a CCD camera outside the chamber is used to the monitor the gap between the source and the cantilever.

Experimental Results

Figure 6 shows the distance versus time for an initial distance of 32 microns with a period of 6 minutes and 8 seconds. Figure 7 shows the measured and calculated contact times for various values of d_0. Each of these graphs use the models in which the resistance R is proportional to the separation and the following values for the constants:

$$\alpha = 0.035$$
$$I = 5\ pA$$
$$A = 10^{-4}\ m^2$$
$$k = 0.16\ N/m$$
$$\rho = 10^{21}\ A/m$$

We have also seen the qualitative behavior derived in the previous sections, that is, we've seen beams stop before reaching the source, but this prediction remains to be verified quantitatively.

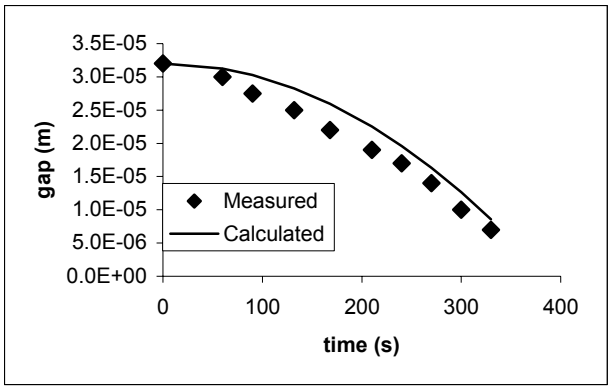

Figure 6: Measured gap for the prototype oscillator

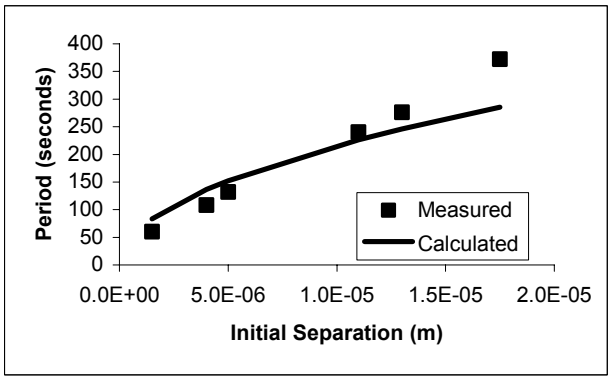

Figure 7: Measured and calculated contact times for the prototype beam.

Conclusions

Radioisotopes offer an attractive power source for MEMS devices for applications that require high power density or long life (without intervention). A direct charging nuclear device with a deformable electrode has been demonstrated, creating a nuclear powered oscillator that can be useful in small-scale applications. The design issues associated with this oscillator have been explored and appropriate dimensionless parameters have been derived to allow simple design. These models have been verified qualitatively with preliminary experiments.

Acknowledgment

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